Sums of reciprocals

Triangular numbers (OEIS A000217)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (n+1)} = 2$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (n+1)} = \log(16) - 2$$

0.772588722239781237668928485832706272302000537441021016482...

Square numbers (OEIS A000290)

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

1.644934066848226436472415166646025189218949901206798437735... (OEIS A013661)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

0.822467033424113218236207583323012594609474950603399218867...(OEIS A072691)

Pentagonal numbers (OEIS A000326)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (3n-1)} = \frac{1}{3} \left(9 \log(3) - \sqrt{3}\ \pi\right)$$

1.482037501770111223591657453125421381658405425375507779634... (OEIS A244641)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (3n-1)} = \frac{2\pi}{\sqrt{3}} - 4 \log(2)$$

0.855010006228654463519228029451605192266131958744460134657...

Hexagonal numbers (OEIS A000384)

$$\sum_{n=1}^{\infty} \frac{1}{n (2n-1)} = \log(4)$$

1.386294361119890618834464242916353136151000268720510508241... (OEIS A016627)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n (2n-1)} = \frac{1}{2} (\pi - \log(4))$$

0.877649146234951309814089570181574874023084565327297656366...

Heptagonal numbers (OEIS A000566)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (5n-3)} = \frac{\sqrt{2}\,(\sqrt{5}-1)\,\pi + \sqrt{5+\sqrt{5}}\,\left(\log(3125) + \sqrt{5}\,\left(\log\left(5-\sqrt{5}\right) - \log\left(5+\sqrt{5}\right)\right)\right)}{6\sqrt{5+\sqrt{5}}}$$

1.322779253122388856749442261310084016522801173713924372285... (OEIS A244639)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (5n-3)} = -\left(-8\sqrt{2}\,\pi + 10\sqrt{5-\sqrt{5}}\,\log(2) + 10\sqrt{5\left(5-\sqrt{5}\right)}\,\log(2) + 5\sqrt{5-\sqrt{5}}\,\log\left(\frac{1}{8}\left(5-\sqrt{5}\right)\right) + \sqrt{5\left(5-\sqrt{5}\right)}\,\log\left(\frac{1}{8}\left(5-\sqrt{5}\right)\right) + 4\sqrt{5-\sqrt{5}}\,\log\left(\sqrt{5}-1\right) - 6\sqrt{5-\sqrt{5}}\,\log\left(1+\sqrt{5}\right) - 2\sqrt{5\left(5-\sqrt{5}\right)}\,\log\left(1+\sqrt{5}\right) - 5\sqrt{5-\sqrt{5}}\,\log\left(\frac{1}{8}\left(5+\sqrt{5}\right)\right) - \sqrt{5\left(5-\sqrt{5}\right)}\,\log\left(\frac{1}{8}\left(5+\sqrt{5}\right)\right)\right) \Big/ \left(6\sqrt{5-\sqrt{5}}\,\left(1+\sqrt{5}\right)\right)$$

0.894240660591454353988045491657380341898817939316881429800...

Octagonal numbers (OEIS A000567)

$$\sum_{n=1}^{\infty} \frac{1}{n (3n-2)} = \frac{1}{12}\left(\sqrt{3}\,\pi + 9\log(3)\right)$$

1.277409057559636731194953492102433211556634480390247232693... (OEIS A244645)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n (3n-2)} = \frac{\pi}{2\sqrt{3}}$$

0.906899682117108925297039128821077866142033124046370287784... (OEIS A093766)

9-gonal numbers (OEIS A001106)

$$\sum_{n=1}^{\infty} \frac{1}{n\left(\frac{1}{2}(7n-5)\right)} = \frac{1}{5}\left(\log(196) + 4\left(\cos\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{3\pi}{14}\right)\right) + \log\left(\sin\left(\frac{\pi}{7}\right)\right)\sin\left(\frac{\pi}{14}\right) + \log\left(\sec\left(\frac{\pi}{14}\right)\right)\sin\left(\frac{3\pi}{14}\right)\right) + \pi\tan\left(\frac{3\pi}{14}\right)\right)$$

1.243320926153712989206607739631014282135844101030099624 41… OEIS A244646)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2}n(7n-5)} = -\frac{1}{10}\tan\left(\frac{\pi}{7}\right)\left(-\pi - \pi\cot^2\left(\frac{\pi}{7}\right) + 6\log(2)\cot\left(\frac{\pi}{7}\right) - 4\cos\left(\frac{\pi}{7}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{\pi}{14}\right)\right) + 4\cos\left(\frac{\pi}{7}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{3\pi}{14}\right)\right) + 4\sin\left(\frac{\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\sin\left(\frac{\pi}{7}\right)\right) + 4\sin\left(\frac{3\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\sin\left(\frac{\pi}{14}\right)\right) + 4\sin\left(\frac{3\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\sin\left(\frac{\pi}{7}\right)\right) - 4\cos\left(\frac{\pi}{7}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\sin\left(\frac{3\pi}{14}\right)\right) - 4\sin\left(\frac{\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{\pi}{7}\right)\right) - 4\sin\left(\frac{\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{3\pi}{14}\right)\right) - 4\sin\left(\frac{3\pi}{14}\right)\cot\left(\frac{\pi}{7}\right)\log\left(\cos\left(\frac{\pi}{14}\right)\right)\right)$$

0.916867160327454898436807848376235910114858551341856393639...

10-gonal numbers (OEIS A001107)

$$\sum_{n=1}^{\infty} \frac{1}{n(4n-3)} = \frac{\pi}{6} + \log(2)$$

1.216745956158244182494339352004760382108361700922772890949... (OEIS A244647)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(4n-3)} = \frac{1}{6}\left(\sqrt{2}\,\pi - \log(4) + 2\sqrt{2}\,\log\left(\cot\left(\frac{\pi}{8}\right)\right)\right)$$

0.924914922933232946959582844691102095511067111673574787739...

11-gonal numbers (OEIS A051682)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (9n-7)} = \frac{1}{7}\left(\log(243) + \pi \cot\left(\frac{2\pi}{9}\right) - 4\sin\left(\frac{\pi}{18}\right)\log\left(\sin\left(\frac{\pi}{9}\right)\right) - 4\cos\left(\frac{2\pi}{9}\right)\log\left(\cos\left(\frac{\pi}{18}\right)\right) + 4\cos\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{2\pi}{9}\right)\right)\right)$$

1.19543411652962797435249923469849935488468262708465806238 6... (OEIS A244648)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (9n-7)} =$$
$$-\frac{1}{14}\tan\left(\frac{\pi}{9}\right)\left(-\pi - \pi\cot^2\left(\frac{\pi}{9}\right) + 8\log(2)\cot\left(\frac{\pi}{9}\right) - 4\cos\left(\frac{\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\cos\left(\frac{\pi}{18}\right)\right) - 4\cos\left(\frac{2\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\cos\left(\frac{\pi}{18}\right)\right) - 4\cos\left(\frac{\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\cos\left(\frac{2\pi}{9}\right)\right) - 4\sin\left(\frac{\pi}{18}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{\pi}{9}\right)\right) + 4\sin\left(\frac{\pi}{18}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{2\pi}{9}\right)\right) + 4\cos\left(\frac{2\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{\pi}{18}\right)\right) + 4\cos\left(\frac{2\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{\pi}{9}\right)\right) + 4\cos\left(\frac{\pi}{9}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\sin\left(\frac{2\pi}{9}\right)\right) + 4\sin\left(\frac{\pi}{18}\right)\cot\left(\frac{\pi}{9}\right)\log\left(\cos\left(\frac{\pi}{9}\right)\right)\right)$$

0.93154681235906014957972296425405587598647863003646520790 8...

12-gonal numbers (OEIS A051624)

$$\sum_{n=1}^{\infty} \frac{1}{n(5n-4)} = \frac{\left(\sqrt{2}+\sqrt{10}\right)\pi + \sqrt{5-\sqrt{5}}\left(\log(3125) + \sqrt{5}\left(\log\left(5+\sqrt{5}\right) - \log\left(5-\sqrt{5}\right)\right)\right)}{16\sqrt{5-\sqrt{5}}}$$

1.17795605792266385873517396809188741844585723456667980 2... (OEIS A244649)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(5n-4)} = -\left(-8\sqrt{2}\,\pi + 2\sqrt{5+\sqrt{5}}\,\log(2) - 2\sqrt{5\left(5+\sqrt{5}\right)}\,\log(2) + 5\sqrt{5+\sqrt{5}}\,\log\left(\frac{1}{8}\left(5-\sqrt{5}\right)\right) - \sqrt{5\left(5+\sqrt{5}\right)}\,\log\left(\frac{1}{8}\left(5-\sqrt{5}\right)\right) + 4\sqrt{5+\sqrt{5}}\,\log\left(\sqrt{5}-1\right) - 6\sqrt{5+\sqrt{5}}\,\log\left(1+\sqrt{5}\right) + 2\sqrt{5\left(5+\sqrt{5}\right)}\,\log\left(1+\sqrt{5}\right) - 5\sqrt{5+\sqrt{5}}\,\log\left(\frac{1}{8}\left(5+\sqrt{5}\right)\right) + \sqrt{5\left(5+\sqrt{5}\right)}\,\log\left(\frac{1}{8}\left(5+\sqrt{5}\right)\right)\right) \Big/ \left(16\left(\sqrt{5}-1\right)\sqrt{5+\sqrt{5}}\right)$$

0.93710517067474947019663600341093004082876350487925890047 7...

k-gonal numbers

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n\left((k-2) n-(k-4)\right)} = \frac{2\left(-\psi^{(0)}\left(\frac{2}{k-2}\right)-\gamma\right)}{k-4} \approx \frac{2\left(-\psi^{(0)}\left(\frac{2}{k-2}\right)-0.577216\right)}{k-4}$$

when $k \neq 2$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n\left((k-2) n-(k-4)\right)} = -\frac{2\left(\log(2)-\Phi\left(-1, 1, \frac{2}{k-2}\right)\right)}{k-4} \approx -\frac{2\left(0.693147-\Phi\left(-1, 1, \frac{2}{k-2}\right)\right)}{k-4} \quad \text{when} \quad k \neq 2$$

Central polygonal numbers (the Lazy Caterer's sequence): n*(n + 1)/2 + 1 (OEIS A000124)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{1}{2}(n+1) n+1} = \frac{2\pi \tanh\left(\frac{\sqrt{7}\,\pi}{2}\right)}{\sqrt{7}}$$

2.373654675440107764321686122237432451913805909406712029673... (OEIS A226985)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{1}{2}(n+1) n+1} = -\frac{2i\left(\Phi\left(-1, 1, \frac{1}{2}\left(1-i\sqrt{7}\right)\right)-\Phi\left(-1, 1, \frac{1}{2}\left(1+i\sqrt{7}\right)\right)\right)}{\sqrt{7}}$$

0.661570192073585112044573892846079395217642466589556979869... (OEIS A228918)

Centered square numbers (OEIS A001844)

$$\sum_{n=0}^{\infty} \frac{1}{2(n+1) n+1} = \frac{1}{2}\pi \tanh\left(\frac{\pi}{2}\right)$$

1.440659519977514592658932502913981712525297084673658690821... (OEIS A228048)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2(n+1) n+1} = \frac{1}{4} i\left(\psi^{(0)}\left(\frac{1}{4}-\frac{i}{4}\right)-\psi^{(0)}\left(\frac{1}{4}+\frac{i}{4}\right)-\psi^{(0)}\left(\frac{3}{4}-\frac{i}{4}\right)+\psi^{(0)}\left(\frac{3}{4}+\frac{i}{4}\right)\right)$$

0.851682256143646274970601831629215259535073105987401689932...

Centered octahedral numbers (OEIS A001845)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{1}{3}(2n+1)(2n^2+2n+3)} = \frac{3}{10}\left(-2\psi^{(0)}\left(\frac{1}{2}\right) + \psi^{(0)}\left(\frac{1}{2}\left(1-i\sqrt{5}\right)\right) + \psi^{(0)}\left(\frac{1}{2}\left(1+i\sqrt{5}\right)\right)\right)$$

1.220983669094861197409059825776386561271974244249611357597...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{1}{3}(2n+1)(2n^2+2n+3)} = \frac{3}{20}\left(-2\psi^{(0)}\left(\frac{1}{4}\right) + 2\psi^{(0)}\left(\frac{3}{4}\right) + \psi^{(0)}\left(\frac{1}{4}\left(1-i\sqrt{5}\right)\right) - \psi^{(0)}\left(\frac{1}{4}\left(3-i\sqrt{5}\right)\right) + \psi^{(0)}\left(\frac{1}{4}\left(1+i\sqrt{5}\right)\right) - \psi^{(0)}\left(\frac{1}{4}\left(3+i\sqrt{5}\right)\right)\right)$$

0.886308906429289502961452329110242136863794676312911818407...

Centered hexagonal numbers (OEIS A003215)

$$\sum_{n=0}^{\infty} \frac{1}{3(n+1)n+1} = \frac{\pi \tanh\left(\frac{\pi}{2\sqrt{3}}\right)}{\sqrt{3}}$$

1.305284153013581141197872587423877835873614184392733587547...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3(n+1)n+1} = -\frac{i\left(\Phi\left(-1, 1, \frac{1}{6}\left(3-i\sqrt{3}\right)\right) - \Phi\left(-1, 1, \frac{1}{6}\left(3+i\sqrt{3}\right)\right)\right)}{\sqrt{3}}$$

0.892680016923400520288523637943378995995784389336095890686...

Centered 12-gonal numbers (OEIS A003154)

$$\sum_{n=0}^{\infty} \frac{1}{6(n-1)n+1} = \frac{1}{6}\left(6+\sqrt{3}\ \pi \tan\left(\frac{\pi}{2\sqrt{3}}\right)\right)$$

2.159173319632174695078566624067437876844635511955939553902...

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{6(n-1)n+1} = \frac{\Phi\left(-1, 1, \frac{1}{6}\left(-3+\sqrt{3}\right)\right) - \Phi\left(-1, 1, \frac{1}{6}\left(-3-\sqrt{3}\right)\right)}{2\sqrt{3}}$$

1.941419057696364686338733077639752815749991058883684283568...

Square pyramidal numbers (OEIS A000330)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{6}n(n+1)(2n+1)} = 6(3-4\log(2))$$

1.364467666561312573986429085003762366187996775353873901103... (OEIS A159354)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{6}n(n+1)(2n+1)} = 6(\pi-3)$$

0.849555921538759430775860299677017305183016396250634925849...

Pentagonal pyramidal numbers (OEIS A002411)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n^2 (n+1)} = \frac{1}{3}\left(\pi^2 - 6\right)$$

1.28986813369645287294483033329205037843789980241359687547 1...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n^2 (n+1)} = 2 + \frac{\pi^2}{6} - 4\log(2)$$

0.87234534460844519880348668081331891691694936376577742125 2...

Hexagonal pyramidal numbers (OEIS A002412)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{6} n (n+1)(4n-1)} = -\frac{6}{5}(1 + 2\pi - 4\log(8))$$

1.24149703144770868329779842912693565821399537628742168899 7...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{6} n (n+1)(4n-1)} = -\frac{6}{5}\tan\left(\frac{\pi}{8}\right)\left(-\pi - \pi\cot^2\left(\frac{\pi}{8}\right) - \cot\left(\frac{\pi}{8}\right) + 6\log(2)\cot\left(\frac{\pi}{8}\right) + 4\sqrt{2}\,\cot\left(\frac{\pi}{8}\right)\log\left(\cos\left(\frac{\pi}{8}\right)\right) - 4\sqrt{2}\,\cot\left(\frac{\pi}{8}\right)\log\left(\sin\left(\frac{\pi}{8}\right)\right)\right)$$

0.88929704620225983645145033124134076108481608858077307790 8...

Heptagonal pyramidal numbers (OEIS A002413)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{6} n (n+1) (5n-2)} =$$

$$6\left(\frac{1-\gamma}{7} + \frac{2}{7}(\gamma-1) - \frac{\gamma}{2} - \frac{5}{14}\left(-\gamma + \frac{1}{8}\sqrt{\frac{5}{\frac{5}{8}+\frac{\sqrt{5}}{8}}}\,\pi - \frac{\pi}{8\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}} - \log(10) - \frac{1}{4}\log\left(\frac{1}{8}(5-\sqrt{5})\right) - \frac{1}{4}\sqrt{5}\log\left(\frac{1}{8}(5-\sqrt{5})\right) - \frac{1}{4}\log\left(\frac{1}{8}(5+\sqrt{5})\right) + \frac{1}{4}\sqrt{5}\log\left(\frac{1}{8}(5+\sqrt{5})\right)\right)\right)$$

1.207293319267837616044611125836777522432231968749322020273...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{6} n (n+1) (5n-2)} =$$

$$-\left(3\left(-8\sqrt{5-\sqrt{5}} - 8\sqrt{5(5-\sqrt{5})} - 40\sqrt{2}\,\pi + 6\sqrt{5-\sqrt{5}}\log(2) + 6\sqrt{5(5-\sqrt{5})}\log(2) - 25\sqrt{5-\sqrt{5}}\log\left(\frac{1}{8}(5-\sqrt{5})\right) - 5\sqrt{5(5-\sqrt{5})}\log\left(\frac{1}{8}(5-\sqrt{5})\right) - 20\sqrt{5-\sqrt{5}}\log(\sqrt{5}-1) + 30\sqrt{5-\sqrt{5}}\log(1+\sqrt{5}) + 10\sqrt{5(5-\sqrt{5})}\log(1+\sqrt{5}) + 25\sqrt{5-\sqrt{5}}\log\left(\frac{1}{8}(5+\sqrt{5})\right) + 5\sqrt{5(5-\sqrt{5})}\log\left(\frac{1}{8}(5+\sqrt{5})\right)\right)\Big/\left(28\sqrt{5-\sqrt{5}}\,(1+\sqrt{5})\right)\right)$$

0.902341934440919373933387039606118430177086408941814662557...

Octagonal pyramidal numbers (OEIS A002414)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (n+1) (2n-1)} = \frac{2}{3}(\log(16)-1)$$

1.181725814826520825112618990555137514868000358294014010988...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (n+1) (2n-1)} = \frac{2}{3}(1+\pi-\log(16))$$

0.912669287566674667195809931631197741263445907956056536328...

4-dimensional pyramidal numbers: (3n+1)*C(n+2,3)/4 (OEIS A001296)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{4}(3n+1)\binom{n+2}{3}} = \frac{6}{5}\left(47 - 3\sqrt{3}\,\pi - 27\log(3)\right)$$

1.215928713423693212378009141174885260753390447141319547641...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{4}(3n+1)\binom{n+2}{3}} = \frac{6}{5}\left(-43 + 6\sqrt{3}\,\pi + 16\log(2)\right)$$

0.886492134210055513642947097067553924385433538520097311426...

4-dimensional pyramidal numbers: n^2*(n^2 - 1)/12 (OEIS A002415)

$$\sum_{n=2}^{\infty} \frac{1}{\frac{1}{12}n^2\left(n^2-1\right)} = 21 - 2\pi^2$$

1.260791197821282762331018000247697729372601185518418747173...

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\frac{1}{12}n^2\left(n^2-1\right)} = \pi^2 - 9$$

0.869604401089358618834490999876151135313699407240790626413...

3-dimensional analog of centered polygonal numbers: C(n,1) + C(n,2) + C(n,3) (OEIS A004006)

$$\sum_{n=1}^{\infty} \frac{1}{\binom{n}{1}+\binom{n}{2}+\binom{n}{3}} = \frac{3}{5}\left(2\gamma + \psi^{(0)}\left(1 - i\sqrt{5}\right) + \psi^{(0)}\left(1 + i\sqrt{5}\right)\right)$$

1.678772955583445210628626183434897224807309197289684533058...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\binom{n}{1}+\binom{n}{2}+\binom{n}{3}} = -\frac{3}{5}\left(\Phi\left(-1, 1, 1 - i\sqrt{5}\right) + \Phi\left(-1, 1, 1 + i\sqrt{5}\right) - \log(4)\right)$$

0.763194382606277184189493468117875897736639291209538182136...

Tetrahedral numbers (OEIS A000292)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{6}n(n+1)(n+2)} = \frac{3}{2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{6}n(n+1)(n+2)} = \log(4096) - \frac{15}{2}$$

0.817766166719343713006785457498118816906001612323063049448... (OEIS A242024)

Octahedral numbers (OEIS A005900)

$$\sum_{n=1}^{\infty}\frac{1}{\frac{1}{3}n\left(2n^2+1\right)}=\frac{3}{2}\left(2\gamma+\psi^{(0)}\left(\frac{1}{2}\left(2-i\sqrt{2}\right)\right)+\psi^{(0)}\left(\frac{1}{2}\left(2+i\sqrt{2}\right)\right)\right)$$

1.278185159090946179540390948367571338423901536851402020170... (OEIS A175577)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\frac{1}{3}n\left(2n^2+1\right)}=-\frac{3}{2}\left(\Phi\left(-1,\,1,\,1-\frac{i}{\sqrt{2}}\right)+\Phi\left(-1,\,1,\,1+\frac{i}{\sqrt{2}}\right)-\log(4)\right)$$

0.870763981156382431002143496424370454319407843702627413073...

Cubic numbers (OEIS A000578)

$$\sum_{n=1}^{\infty}\frac{1}{n^3}=\zeta(3)$$

1.202056903159594285399738161511449990764986292340498881792... (OEIS A002117)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3}=\frac{3\,\zeta(3)}{4}$$

0.901542677369695714049803621133587493073739719255374161344... (OEIS A197070)

Icosahedral numbers (OEIS A006564)

$$\sum_{n=1}^{\infty}\frac{1}{\frac{1}{2}n\left(5n^2-5n+2\right)}=2\left(\frac{\gamma}{2}-\frac{\psi^{(0)}\left(\frac{1}{10}\left(5-i\sqrt{15}\right)\right)}{7+2\left(-5+i\sqrt{15}\right)+\frac{3}{20}\left(-5+i\sqrt{15}\right)^2}-\frac{\psi^{(0)}\left(\frac{1}{10}\left(5+i\sqrt{15}\right)\right)}{7+2\left(-5-i\sqrt{15}\right)+\frac{3}{20}\left(-5-i\sqrt{15}\right)^2}\right)$$

1.123565966899251887573937579015879645353811416485504980606...(OEIS A175578)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\frac{1}{2}n\left(5n^2-5n+2\right)}=\frac{1}{6}\left(\left(-3-i\sqrt{15}\right)\Phi\left(-1,\,1,\,\frac{1}{2}-\frac{1}{2}i\sqrt{\frac{3}{5}}\right)+i\left(\sqrt{15}+3i\right)\Phi\left(-1,\,1,\,\frac{1}{2}+\frac{1}{2}i\sqrt{\frac{3}{5}}\right)+\log(64)\right)$$

0.931984162648342178506866443403568986062988389229066468616...

Dodecahedral numbers (OEIS A006566)

$$\sum_{n=1}^{\infty}\frac{1}{\frac{1}{2}n\left(3n-1\right)\left(3n-2\right)}=\frac{1}{2}\left(\sqrt{3}\,\pi-3\log(3)\right)$$

1.072780613349162238798249531079445041454863535404986685752...

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\frac{1}{2}n\left(3n-1\right)\left(3n-2\right)}=\log(16)-\frac{\pi}{\sqrt{3}}$$

0.958789358005563387074850228190550540017934289348280440912...

n^2 + 1 (OEIS A002522)

$$\sum_{n=0}^{\infty} \frac{1}{n^2+1} = \frac{1}{2}(1+\pi\coth(\pi))$$

2.076674047468581174134050794750000490445656266403816665575... (OEIS A113319)

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^2+1} = \frac{1}{4} i \left(\psi^{(0)}\left(\frac{1}{2}-\frac{i}{2}\right) - \psi^{(0)}\left(\frac{1}{2}+\frac{i}{2}\right) - \psi^{(0)}\left(1-\frac{i}{2}\right) + \psi^{(0)}\left(1+\frac{i}{2}\right)\right)$$

0.363985472508933418524881708163981222079640818269842025246...

n^2 - n + 1 (OEIS A002061)

$$\sum_{n=0}^{\infty} \frac{1}{n^2-n+1} = \frac{i\left(\psi^{(0)}\left(-\sqrt[3]{-1}\right) - \psi^{(0)}\left((-1)^{2/3}\right)\right)}{\sqrt{3}}$$

2.798147280562690180905820124337422932920324634371839143386...

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^2-n+1} = \frac{i\left(\Phi\left(-1,1,-\sqrt[3]{-1}\right) - \Phi\left(-1,1,(-1)^{2/3}\right)\right)}{\sqrt{3}}$$

1.761310204001103486388731278677838411604882715779726095226...

n^4 + 1 (OEIS A002523)

$$\sum_{n=0}^{\infty} \frac{1}{n^4+1} = \frac{1}{4}\left(2+\sqrt[4]{-1}\,\pi\left(\cot\left(\sqrt[4]{-1}\,\pi\right) + i\cot\left((-1)^{3/4}\pi\right)\right)\right)$$

1.578477579667136838318022193245719235046672217327291327587...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^4+1} =$$

$$\left(\frac{1}{16}+\frac{i}{16}\right)\left((4-4i)+\sqrt{2}\,\pi\tan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\pi}{\sqrt{2}}\right) + (1-i)\sqrt[4]{-1}\,\pi\cot\left(\frac{1}{2}\sqrt[4]{-1}\,\pi\right) + (1-i)(-1)^{3/4}\pi\cot\left(\frac{1}{2}(-1)^{3/4}\pi\right) - \sqrt{2}\,\pi\tanh\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\pi}{\sqrt{2}}\right)\right)$$

0.549428148719873179229293598058766115032173518319778254145...

4th powers (OEIS A000583)

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

1.082323233711138191516003696541167902774750951918726907682... (OEIS A013662)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}$$

0.947032829497245917576503234473521914927907082928886044222... (OEIS A267315)

5th powers (OEIS A000584)

$$\sum_{n=1}^{\infty} \frac{1}{n^5} = \zeta(5)$$

1.036927755143369926331365486457034168057080919501912811974... (OEIS A013663)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} = \frac{15\,\zeta(5)}{16}$$

0.972119770446909305935655143553469532553513362033043261225... (OEIS A267316)

6th powers (OEIS A001014)

$$\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$$

1.017343061984449139714517929790920527901817490032853561842... (OEIS A013664)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6} = \frac{31\,\pi^6}{30\,240}$$

0.985551091297435104098439244484954261404885693469326888034... (A275703)

7th powers (OEIS A001015)

$$\sum_{n=1}^{\infty} \frac{1}{n^7} = \zeta(7)$$

1.008349277381922826839797549849796759599863560565238706417... (OEIS A013665)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^7} = \frac{63\,\zeta(7)}{64}$$

0.992593819922830282670425713133393685231115692431406851629... (OEIS A275710)

8th powers (OEIS A001016)

$$\sum_{n=1}^{\infty} \frac{1}{n^8} = \frac{\pi^8}{9450}$$

1.004077356197944339378685238508652465258960790649850020329... (OEIS A013666)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^8} = \frac{127\,\pi^8}{1\,209\,600}$$

0.996233001852647899227289260082803617874125159472898067045...

Squares of primes (OEIS A001248)

$$\sum_{n=1}^{\infty} \frac{1}{(p_n)^2} = P(2)$$

0.452247420041065498506543364832247934173231343239892421736... (OEIS A085548)

Cubes of primes (OEIS A030078)

$$\sum_{n=1}^{\infty} \frac{1}{(p_n)^3} = P(3)$$

0.174762639299443536423113314665706700975412121926149289888... (OEIS A085541)

Oblong numbers (OEIS A002378)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\,(n+1)} = \log(4) - 1$$

0.386294361119890618834464242916353136151000268720510508241...

Odd numbers (OEISA005408)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2\,n+1} = \frac{\pi}{4}$$

0.785398163397448309615660845819875721049292349843776455243... (OEIS A003881)

Odd squares (OEIS A016754)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n+1)^2} = \frac{\pi^2}{8}$$

1.233700550136169827354311374984518891914212425905098828301... (OEIS A111003)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n+1)^2} = C$$

0.915965594177219015054603514932384110774149374281672134266... (OEIS A006752)

Odd cubes (OEIS A016755)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n+1)^3} = \frac{7\,\zeta(3)}{8}$$

1.051799790264644999724770891322518741919363005797936521568... (OEIS A233091)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n+1)^3} = \frac{\pi^3}{32}$$

0.968946146259369380483634845846918600069540267683909615442... (OEIS A153071)

(2n + 1)^4 (OEIS A016756)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n+1)^4} = \frac{\pi^4}{96}$$

1.014678031604192054546253465507344908851329017423806475952...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n+1)^4} = \frac{1}{256}\left(\zeta\left(4, \frac{1}{4}\right) - \zeta\left(4, \frac{3}{4}\right)\right)$$

0.988944551741105336108422633228377821315860887062733910781... (OEIS A175572)

(2n + 1)^5 (OEIS A016757)

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^5} = \frac{31\,\zeta(5)}{32}$$

1.00452376279513961633510315005251850305297140767478036599...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^5} = \frac{5\,\pi^5}{1536}$$

0.99615782807708806400631936863097528151139552938826494320 7... (OEIS A175571)

(2n + 1)^k

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^k} = 2^{-k}\left(-1+2^k\right)\zeta(k) \quad \text{when} \quad \text{Re}(k) > 1$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^k} = 2^{-2k}\left(\zeta\left(k, \frac{1}{4}\right) - \zeta\left(k, \frac{3}{4}\right)\right) \approx 2^{-2k}\left(\zeta\left(k, \frac{1}{4}\right) - \zeta(k, 0.75)\right)$$

when $\text{Re}(k) > 1$

2^n - 1 (OEIS A000225)

$$\sum_{n=1}^{\infty} \frac{1}{2^n - 1} = 1 - \frac{\psi_{\frac{1}{2}}^{(0)}(1)}{\log(2)}$$

1.60669515241529176378330152319092458048057967150575643577 8... (OEIS A065442)

2^n + 1 (OEIS A000051)

$$\sum_{n=0}^{\infty} \frac{1}{2^n + 1} = -1 + \frac{\psi_{\frac{1}{2}}^{(0)}\left(-\frac{i\,\pi}{\log(2)}\right)}{\log(2)}$$

1.26449978034844420919131974725549848255769699885752562656 6...

3^n - 1 (OEIS A024023)

$$\sum_{n=1}^{\infty} \frac{1}{3^n - 1} = \frac{\log\left(\frac{3}{2}\right) - \psi_{\frac{1}{3}}^{(0)}(1)}{\log(3)}$$

0.68215350260523806676126318622662400964919024832690341922 8... (OEIS A214369)

3^n + 1 (OEIS A034472)

$$\sum_{n=0}^{\infty} \frac{1}{3^n + 1} = \frac{-\log\left(\frac{3}{2}\right) + \psi_{\frac{1}{3}}^{(0)}\left(-\frac{i\,\pi}{\log(3)}\right)}{\log(3)}$$

0.90406326728086180804380623748844590537046495419126420012 6...

4^n − 1 (OEIS A024036)

$$\sum_{n=1}^{\infty}\frac{1}{4^n-1}=\frac{\log\left(\frac{4}{3}\right)-\psi_{\frac{1}{4}}^{(0)}(1)}{\log(4)}$$

0.421097686033423777295990887967713048961441336324115404605... (OEIS A248721)

4^n + 1 (OEIS A052539)

$$\sum_{n=0}^{\infty}\frac{1}{4^n+1}=\frac{-\log\left(\frac{4}{3}\right)+\psi_{\frac{1}{4}}^{(0)}\left(-\frac{i\pi}{\log(4)}\right)}{\log(4)}$$

0.779400262405960144170530146934068249982019850340830775541...

5^n − 1 (OEIS A024049)

$$\sum_{n=1}^{\infty}\frac{1}{5^n-1}=\frac{\log\left(\frac{5}{4}\right)-\psi_{\frac{1}{5}}^{(0)}(1)}{\log(5)}$$

0.301733853597972457948162159393991192623009431517157720395... (OEIS A248722)

5^n + 1 (OEIS A034474)

$$\sum_{n=0}^{\infty}\frac{1}{5^n+1}=\frac{-\log\left(\frac{5}{4}\right)+\psi_{\frac{1}{5}}^{(0)}\left(-\frac{i\pi}{\log(5)}\right)}{\log(5)}$$

0.715062050520525025911840094339842060152355557493907738974...

k^n − 1

$$\sum_{n=1}^{\infty}\frac{1}{k^n-1}=-\frac{2\log(k-1)+2\log\left(\frac{1}{k}\right)+\log(k)+2\,\psi_k^{(0)}\left(-\frac{\log\left(\frac{1}{k}\right)}{\log(k)}\right)}{2\log(k)}\quad\text{when}\quad|k|>1$$

k^n + 1

$$\sum_{n=1}^{\infty}\frac{1}{k^n+1}=\frac{2\log(k-1)+2\log\left(-\frac{1}{k}\right)+\log(k)+2\,\psi_k^{(0)}\left(-\frac{\log\left(-\frac{1}{k}\right)}{\log(k)}\right)}{2\log(k)}\quad\text{when}\quad|k|>1$$

n*(n + 3)/2 (OEIS A000096)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2} n (n+3)} = \frac{11}{9}$$

1.22222222222222222222222222222222222222222222222222222222...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{2} n (n+3)} = \frac{4 \log(2)}{3} - \frac{5}{9}$$

0.36864068519103819033408727305534653521177795692478478327 2...

2*n*(n + 1)*(2*n + 1)/3 (OEIS A002492)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{2}{3} n (n+1) (2n+1)} = \frac{9}{2} - 6 \log(2)$$

0.34111691664032814349660727125094059154699919383846847527 5...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{2}{3} n (n+1) (2n+1)} = \frac{3}{2} (\pi - 3)$$

0.21238898038468985769396507491925432629575409906265873146 2...

Sum of fourth powers (OEIS A000538)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{30} n (n+1) (2n+1) \left(3n^2+3n-1\right)} = \frac{30}{7} \left(7 - 14\gamma - 8\psi^{(0)}\left(\frac{3}{2}\right) - 3\psi^{(0)}\left(\frac{1}{6}\left(9-\sqrt{21}\right)\right) - 3\psi^{(0)}\left(\frac{1}{6}\left(9+\sqrt{21}\right)\right)\right)$$

1.07383121333922776218975880800626342971878462740200259904 0...

Doubly triangular numbers (OEIS A002817)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{8} n (n+1) \left(n^2+n+2\right)} = -\frac{16\left(2\sqrt{7}\,\pi \tanh\left(\frac{\sqrt{7}\,\pi}{2}\right) - 21\right)}{7\left(\sqrt{7}-i\right)\left(\sqrt{7}+i\right)}$$

1.25269064911978447135662775552513509617238818118657594065 3...

(n*(n + 1)/2)^2 (OEIS A000537)

$$\sum_{n=1}^{\infty} \frac{1}{\left(\frac{1}{2} n (n+1)\right)^2} = \frac{4}{3} \left(\pi^2 - 9\right)$$

1.15947253478581149177932133316820151375159920965438750188 4...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\left(\frac{1}{2} n (n+1)\right)^2} = 12 - 16 \log(2)$$

0.90964511104087504932428605666917491079199785023591593406 9...

n*2^(n-1) (OEIS A001787)

$$\sum_{n=1}^{\infty} \frac{1}{n\,2^{n-1}} = \log(4)$$

1.38629436111989061883446424291635313615100026872051050824 1... (OEIS A016627)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\,2^{n-1}} = \log\left(\frac{9}{4}\right)$$

0.81093021621632876395602623092869827314398084692498839522 8...

n*(n + 1)*2^(n-2) (OEIS A001788)

$$\sum_{n=1}^{\infty} \frac{1}{n\,(n+1)\,2^{n-2}} = 4 - 4\log(2)$$

1.22741127776021876233107151416729372769799946255897898351 7...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\,(n+1)\,2^{n-2}} = 12\log\left(\frac{3}{2}\right) - 4$$

0.86558129729797258373615738557218963886388508154993037136 8...

(3^n - 1)/2 (OEIS A003462)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{2}\,(3^n - 1)} = \frac{\log\left(\frac{9}{4}\right) - 2\,\psi_{\frac{1}{3}}^{(0)}(1)}{\log(3)}$$

1.36430700521047613352252637245324801929838049665380683845 6...

n*(4*n^2 - 1)/3 (A000447)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{3}\,n\left(4\,n^2 - 1\right)} = \log(64) - 3$$

1.15888308335967185650339272874905940845300080616153152472 4...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{1}{3}\,n\left(4\,n^2 - 1\right)} = 3 - \log(8)$$

0.92055845832016407174830363562547029577349959691923423763 7...

(4^n - 1)/3 (OEIS A002450)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{1}{3}\,(4^n - 1)} = \frac{3\left(\log\left(\frac{4}{3}\right) - \psi_{\frac{1}{4}}^{(0)}(1)\right)}{\log(4)}$$

1.26329305810027133188797266390313914688432400897234621381 7...

a(n) = 4*a(n-1) - a(n-2), a(0) = 0, a(1) = 1 (OEIS A001353)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{\left(\sqrt{3}+2\right)^n-\left(2-\sqrt{3}\right)^n}{2\sqrt{3}}} = \frac{\sqrt{3}\left(\psi^{(0)}_{2-\sqrt{3}}(1)-\psi^{(0)}_{2-\sqrt{3}}\left(1-\frac{i\pi}{\log\left(2-\sqrt{3}\right)}\right)\right)}{\log\left(2-\sqrt{3}\right)}$$

1.34105981307843022497838320712111103611544442253062141236 6...

a(n) = 4*a(n-1) - a(n-2), a(0) = 1, a(1) = 2 (OEIS A001075)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{1}{2}\left(\left(2-\sqrt{3}\right)^n+\left(\sqrt{3}+2\right)^n\right)} = \frac{i\left(\psi^{(0)}_{2-\sqrt{3}}\left(-\frac{i\pi}{2\log\left(2-\sqrt{3}\right)}\right)-\psi^{(0)}_{2-\sqrt{3}}\left(\frac{i\pi}{2\log\left(2-\sqrt{3}\right)}\right)\right)}{\log\left(2-\sqrt{3}\right)}$$

1.69540151233398412523818469383716945458564282448639702231 9...

a(n) = 4*a(n-1) - a(n-2), a(0) = 2, a(1) = 4 (OEIS A003500)

$$\sum_{n=0}^{\infty} \frac{1}{\left(2-\sqrt{3}\right)^n+\left(\sqrt{3}+2\right)^n} = \frac{i\left(\psi^{(0)}_{2-\sqrt{3}}\left(-\frac{i\pi}{2\log\left(2-\sqrt{3}\right)}\right)-\psi^{(0)}_{2-\sqrt{3}}\left(\frac{i\pi}{2\log\left(2-\sqrt{3}\right)}\right)\right)}{2\log\left(2-\sqrt{3}\right)}$$

0.84770075616699206261909234691858472729282141224319851115 9...

a(n) = 6*a(n-1) - a(n-2), a(0) = 1, a(1) = 3 (OEIS A001541)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{1}{2}\left(\left(3-2\sqrt{2}\right)^n+\left(2\sqrt{2}+3\right)^n\right)} = \frac{i\left(\psi^{(0)}_{3-2\sqrt{2}}\left(-\frac{i\pi}{2\log\left(3-2\sqrt{2}\right)}\right)-\psi^{(0)}_{3-2\sqrt{2}}\left(\frac{i\pi}{2\log\left(3-2\sqrt{2}\right)}\right)\right)}{\log\left(3-2\sqrt{2}\right)}$$

1.40434991259115783647914720735798105907854051134113427724 5...

Factorial numbers (OEIS A000142)

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e$$

2.71828182845904523536028747135266249775724709369995957496 6... (OEIS A001113)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$$

0.36787944117144232159552377016146086744581113103176783450 7... (OEIS A068985)

Central binomial coefficients (OEIS A000984)

$$\sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}} = \frac{2}{27}\left(18+\sqrt{3}\ \pi\right)$$

1.736399858718715077909795168364923496063125832909497905682... (OEIS A091682)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\binom{2n}{n}} = -\frac{4}{25}\left(\sqrt{5}\ \sinh^{-1}\left(\frac{1}{2}\right)-5\right)$$

0.627836423614398384444226706819757829830171726983884138097...

Catalan numbers (OEIS A000108)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{(2n)!}{n!\,(n+1)!}} = 2+\frac{4\pi}{9\sqrt{3}}$$

2.806133050770763489152923670063180325459584999152329144697... (OEIS A268813)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{(2n)!}{n!\,(n+1)!}} = \frac{14}{25}-\frac{24\,\text{csch}^{-1}(2)}{25\sqrt{5}}$$

0.353403708337278061333072048183709395796206072380660965716...

n*n! (OEIS A001563)

$$\sum_{n=1}^{\infty} \frac{1}{n\,n!} = \text{Ei}(1)-\gamma$$

1.317902151454403894860008844249231837974901245792783992840... (OEIS A229837)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\,n!} = \gamma-\text{Ei}(-1)$$

0.796599599297053134283675865542524080073206629346831806383... (OEIS A239069)

(n!)^2 (OEIS A001044)

$$\sum_{n=0}^{\infty} \frac{1}{(n!)^2} = I_0(2)$$

2.279585302336067267437204440811533353285841102785459054070... (OEIS A070910)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} = J_0(2)$$

0.223890779141235668051827454649948625825154482218607603128... (OEIS A091681)

(n!)^3 (OEIS A000442)

$$\sum_{n=0}^{\infty} \frac{1}{(n!)^3} = {}_0F_2\,(;\,1,\,1;\,1)$$

2.12970254898330641813452361059541346831922074703916930376 2... (OEIS A271574)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^3} = {}_0F_2\,(;\,1,\,1;\,-1)$$

0.12044213230101764656130083969421333998475799178569290659 7...

(2n + 1)! (OEIS A009445)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n+1)!} = \sinh(1)$$

1.17520119364380145688238185059560081515571798133409587022 9... (OEIS A073742)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n+1)!} = \sin(1)$$

0.84147098480789650665250232163029899962256306079837106567 2... (OEIS A049469)

(2n)! (OEIS A010050)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n)!} = \cosh(1)$$

1.54308063481524377847790562075706168260152911236586370473 7... (OEIS A073743)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n)!} = \cos(1)$$

0.54030230586813971740093660744297660373231042061792222767 0... (OEIS A049470)

(3n + 1)! (OEIS A100089)

$$\sum_{n=0}^{\infty} \frac{1}{(3\,n+1)!} = \frac{e}{3} + \frac{2\sin\left(\frac{1}{6}\left(3\sqrt{3}-\pi\right)\right)}{3\sqrt{e}}$$

1.04186535509890984630133661502152738769708357172416345954 5... (OEIS A143820)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(3\,n+1)!} = \frac{2\,e^{3/2}\sin\left(\frac{1}{6}\left(3\sqrt{3}+\pi\right)\right)-1}{3\,e}$$

0.95853147061909644370176312455081091735785106598159557204 5...

(3n)! (OEIS A100732)

$$\sum_{n=0}^{\infty} \frac{1}{(3n)!} = \frac{1}{3}\left(e + \frac{2\cos\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{e}}\right)$$

1.168058313375918525516256929611144747716933295113292516385... (OEIS A143819)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(3n)!} = \frac{1 + 2\,e^{3/2}\cos\left(\frac{\sqrt{3}}{2}\right)}{3e}$$

0.834719468577210962219283239208330070840379051998269767627...

(4n + 1)!

$$\sum_{n=0}^{\infty} \frac{1}{(4n+1)!} = \frac{1}{2}(\sin(1) + \sinh(1))$$

1.008336089225848981767442086112949907389140521066233467951...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)!} = \frac{\cos\left(\frac{1}{\sqrt{2}}\right)\sinh\left(\frac{1}{\sqrt{2}}\right) + \sin\left(\frac{1}{\sqrt{2}}\right)\cosh\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2}}$$

0.991669422238001438325197245331426975599950647350206972134...

(4n)! (OEIS A100733)

$$\sum_{n=0}^{\infty} \frac{1}{(4n)!} = \frac{1}{2}(\cos(1) + \cosh(1))$$

1.041691470341691747939421114100019143166919766491892966203...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n)!} = \cos\left(\frac{1}{\sqrt{2}}\right)\cosh\left(\frac{1}{\sqrt{2}}\right)$$

0.958358132833007016210404460255674995423556794701810169561...

(5n + 1)!

$$\sum_{n=0}^{\infty} \frac{1}{(5n+1)!} = {}_0F_4\left(;\frac{2}{5},\frac{3}{5},\frac{4}{5},\frac{6}{5};\frac{1}{3125}\right)$$

1.001388913941045069123504481336959302473670717401813362652...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(5n+1)!} = {}_0F_4\left(;\frac{2}{5},\frac{3}{5},\frac{4}{5},\frac{6}{5};-\frac{1}{3125}\right)$$

0.998611136163171701799078950894040234096470292153063981152...

(5n)! (OEIS A100734)

$$\sum_{n=0}^{\infty} \frac{1}{(5\,n)!} = {}_0F_4\left(;\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5};\frac{1}{3125}\right)$$

1.008333608907290289976453667354838786071077281579543102003... (OEIS A269296)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(5\,n)!} = {}_0F_4\left(;\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5};-\frac{1}{3125}\right)$$

0.991666942239094190563422908453986205317591525067808393358...

n!*(n - 1)!/2^(n-1) (OEIS A006472)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{n!\,(n-1)!}{2^{n-1}}} = \frac{I_1\left(2\sqrt{2}\right)}{\sqrt{2}}$$

2.394833099273404716522632636436373151968637007091362444726...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{n!\,(n-1)!}{2^{n-1}}} = \frac{J_1\left(2\sqrt{2}\right)}{\sqrt{2}}$$

0.282979986880542502801500701549384643523509876995535827608...

(2n)!/2^n (OEIS A000680)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{(2\,n)!}{2^n}} = \cosh\left(\sqrt{2}\right)$$

2.178183556608570863989222067820125283431294032921656932810...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{(2\,n)!}{2^n}} = \cos\left(\sqrt{2}\right)$$

0.155943694765374473454647978908589641624447250391305356890...

Quadruple factorial numbers (OEIS A001813)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{(2\,n)!}{n!}} = \frac{1}{2}\sqrt[4]{e}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{1}{2}\right)+1$$

1.592296536469326575660415054539062687284616612216987103775...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{(2\,n)!}{n!}} = 1-\frac{\sqrt{\pi}\,\operatorname{erfi}\left(\frac{1}{2}\right)}{2\sqrt[4]{e}}$$

0.575563616497977704065957647510330428903570522640307961848...

Lah numbers (OEIS A001286)

$$\sum_{n=2}^{\infty} \frac{1}{\frac{1}{2}(n-1)\,n!} = -2\,(-\mathrm{Ei}(1) - 2 + e + \gamma)$$

1.19924064599071731899944274579313868043530830418564883574 6...

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\frac{1}{2}(n-1)\,n!} = -2\,\mathrm{Ei}(-1) - \frac{2}{e} + 2\,\gamma$$

0.85744031625122162537630419076212642525479099663012794375 1...

(3/2)^n*Gamma(n+2/3)*Gamma(n+1)/Gamma(2/3) (OEIS A084939)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{\left(\frac{3}{2}\right)^n \Gamma\left(n+\frac{2}{3}\right)\Gamma(n+1)}{\Gamma\left(\frac{2}{3}\right)}} = \sqrt[6]{\frac{2}{3}}\;\Gamma\left(\frac{2}{3}\right) I_{-\frac{1}{3}}\left(2\sqrt{\frac{2}{3}}\right)$$

2.21744631798757752705489153522170199904582262071244215882 5...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{\left(\frac{3}{2}\right)^n \Gamma\left(n+\frac{2}{3}\right)\Gamma(n+1)}{\Gamma\left(\frac{2}{3}\right)}} = \sqrt[6]{\frac{2}{3}}\;\Gamma\left(\frac{2}{3}\right) J_{-\frac{1}{3}}\left(2\sqrt{\frac{2}{3}}\right)$$

0.18406968248376502135859543693212712442737912792686779599 5...

2*3^n*(2*n)!/(n!*(n + 2)!) (OEIS A000168)

$$\sum_{n=0}^{\infty} \frac{1}{2\times 3^n \times \frac{(2n)!}{n!\,(n+2)!}} = \frac{1887}{1331} + \frac{3240\,\csc^{-1}\left(2\sqrt{3}\right)}{1331\sqrt{11}}$$

1.63266502918571641290394782463233521218239160307021767116 2...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2\times 3^n \times \frac{(2n)!}{n!\,(n+2)!}} = \frac{1563}{2197} - \frac{3240\,\mathrm{csch}^{-1}\left(2\sqrt{3}\right)}{2197\sqrt{13}}$$

0.59493241228508219173953447162486252644386562402451135184 8...

2*(3*n)!/((2*n + 1)!*((n + 1)!)) (OEIS A000139)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{2\,(3n)!}{(2n+1)!\,(n+1)!}} = \frac{1}{2}\,{}_3F_2\left(1, \frac{3}{2}, 2; \frac{1}{3}, \frac{2}{3}; \frac{4}{27}\right)$$

2.22620619929126159869598464196604905300083365935835664316 5...

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\frac{2\,(3n)!}{(2n+1)!\,(n+1)!}} = -\frac{1}{2}\,{}_3F_2\left(1, \frac{3}{2}, 2; \frac{1}{3}, \frac{2}{3}; -\frac{4}{27}\right)$$

0.13017916104106597730225079304773397177337232986705193241 8...

3*(2*n)!/((n + 2)!*(n - 1)!) (OEIS A000245)

$$\sum_{n=1}^{\infty} \frac{1}{\frac{3\,(2\,n)!}{(n+2)!\,(n-1)!}} = {}_3F_2\left(1, 1, 4; \frac{3}{2}, 2; \frac{1}{4}\right)$$

1.4960441147881129595672998372959326019250713878999395577 02...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\frac{3\,(2\,n)!}{(n+2)!\,(n-1)!}} = {}_3F_2\left(1, 1, 4; \frac{3}{2}, 2; -\frac{1}{4}\right)$$

0.7505804421206444158524470013593285984641972795773833558 67...

(2*n + 1)!/n! (OEIS A000407)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{(2\,n+1)!}{n!}} = \sqrt[4]{e}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{1}{2}\right)$$

1.1845930729386531513208301090781253745692332244339742075 51...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{(2\,n+1)!}{n!}} = \frac{\sqrt{\pi}\,\operatorname{erfi}\left(\frac{1}{2}\right)}{\sqrt[4]{e}}$$

0.8488727670040445918680847049793391421928589547193840763 02...

(2*n + 1)!/(n!)^2 (OEIS A002457)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{(2\,n+1)!}{(n!)^2}} = \frac{2\,\pi}{3\,\sqrt{3}}$$

1.2091995761561452337293855050947704881893774987284937170 46... (OEIS A248897)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{(2\,n+1)!}{(n!)^2}} = \frac{4\,\sinh^{-1}\left(\frac{1}{2}\right)}{\sqrt{5}}$$

0.8608178819280080777788664659012108508491413650805793095 14...

Double factorial of odd numbers (OEIS A001147)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n-1)!!} = \frac{1}{2}\left(\sqrt{2\,e\,\pi}\,\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)+2\right)$$

2.4106861346424479976908247114191150413234786256251921977 24..

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n-1)!!} = 1-\sqrt{\frac{\pi}{2\,e}}\,\operatorname{erfi}\left(\frac{1}{\sqrt{2}}\right)$$

0.2752215409929236681817720323937838336878670693761825509 27...

Double factorial of even numbers (OEIS A000165)

$$\sum_{n=0}^{\infty} \frac{1}{(2\,n)!!} = \sqrt{e}$$

1.648721270700128146848650787814163571653776100710148011575... (OEIS A019774)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2\,n)!!} = \frac{1}{\sqrt{e}}$$

0.606530659712633423603799534991180453441918135487186955682... (OEIS A092605)

((2*n - 1)!!)^2 (OEIS A001818)

$$\sum_{n=1}^{\infty} \frac{1}{((2\,n-1)!!)^2} = \frac{\pi\,\mathrm{L}_0(1)}{2}$$

1.115647387602343779553464946855446185188573510407208707622...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{((2\,n-1)!!)^2} = \frac{\pi\,\mathrm{H}_0(1)}{2}$$

0.893243740975026168343711800625978646880740228064341655999...

Binomial coefficients C(n,4) (OEIS A000332)

$$\sum_{n=4}^{\infty} \frac{1}{\binom{n}{4}} = \frac{4}{3}$$

1.333333333333333333333333333333333333333333333333333333333333...

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{\binom{n}{4}} = \frac{32}{3}(\log(8) - 2)$$

0.847376444584916568018094553328316845082670966194834798528... (OEIS A242023)

Binomial coefficients C(n,5) (OEIS A000389)

$$\sum_{n=5}^{\infty} \frac{1}{\binom{n}{5}} = \frac{5}{4}$$

$$\sum_{n=5}^{\infty} \frac{(-1)^{n+1}}{\binom{n}{5}} = 80\log(2) - \frac{655}{12}$$

0.868441111462291420045236383320792112706677415487086996321...

Binomial coefficients C(n,6) (OEIS A000579)

$$\sum_{n=6}^{\infty}\frac{1}{\binom{n}{6}}=\frac{6}{5}$$

$$\sum_{n=6}^{\infty}\frac{(-1)^n}{\binom{n}{6}}=192\log(2)-\frac{661}{5}$$

0.884258667509499408108567319969901070496025797169008791170...

Stirling numbers of the first kind (OEIS A000914)

$$\sum_{n=1}^{\infty}\frac{1}{\frac{1}{4}(3n+5)\binom{n+2}{3}}=-\frac{6}{25}\left(64+15\sqrt{3}\ \pi-135\log(3)\right)$$

0.646005019117201214789900493754550921910778594055484020049...

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\frac{1}{4}(3n+5)\binom{n+2}{3}}=\frac{12}{25}\left(-53+15\sqrt{3}\ \pi-40\log(2)\right)$$

0.429640400708155632021233633073573710286228379086295553192...

Apéry numbers: n*C(2n,n) (OEIS A005430)

$$\sum_{n=1}^{\infty}\frac{1}{n\binom{2n}{n}}=\frac{\pi}{3\sqrt{3}}$$

0.604599788078072616864692752547385244094688749364246858523... (OEIS A073010)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\binom{2n}{n}}=\frac{2\sinh^{-1}\left(\frac{1}{2}\right)}{\sqrt{5}}$$

0.430408940964004038889433232950605425424570682540289654757... (OEIS A086466)

Apéry numbers: n^2*C(2n,n) (OEIS A002736)

$$\sum_{n=1}^{\infty}\frac{1}{n^2\binom{2n}{n}}=\frac{\pi^2}{18}$$

0.548311355616075478824138388882008396406316633735599479245... (OEIS A086463)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2\binom{2n}{n}}=2\sinh^{-1}\left(\frac{1}{2}\right)^2$$

0.463129641154388784993858142463065520032812700098597741630... (OEIS A086467)

Apéry numbers: n^3*C(2n,n) (OEIS A005429)

$$\sum_{n=1}^{\infty} \frac{1}{n^3 \binom{2n}{n}} = \frac{1}{2}\,{}_4F_3\left(1, 1, 1, 1; \frac{3}{2}, 2, 2; \frac{1}{4}\right)$$

0.52294619213333510849118518352730354016304459174397784146 5... (OEIS A145438)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3 \binom{2n}{n}} = \frac{2\,\zeta(3)}{5}$$

0.48082276126383771415989526460457999630599451693619955271 6... (OEIS A086468)

C(2n,n-1) (OEIS A001791)

$$\sum_{n=1}^{\infty} \frac{1}{\binom{2n}{n-1}} = \frac{1}{27}\left(9 + 5\sqrt{3}\,\pi\right)$$

1.34099964679678769477448792091230874015781458227374476420 5...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\binom{2n}{n-1}} = \frac{1}{60}\left(48\sqrt{5}\,\sinh^{-1}\left(\frac{1}{2}\right) - 5\,{}_2F_1\left(2, 2; \frac{5}{2}; -\frac{1}{4}\right)\right)$$

0.80257251734960565444520652613084759559439895555640551665 9...

C(2n+1, n+1) (OEIS A001700)

$$\sum_{n=0}^{\infty} \frac{1}{\binom{2n+1}{n+1}} = \frac{2}{27}\left(9 + 2\sqrt{3}\,\pi\right)$$

1.47279971743743015581959033672984699212625166581899581136 4... (OEIS A248179)

$$\sum_{n=0}^{\infty} \frac{(-1)^{n}}{\binom{2n+1}{n+1}} = \frac{2}{25}\left(5 + 4\sqrt{5}\,\sinh^{-1}\left(\frac{1}{2}\right)\right)$$

0.74432715277120323111154658636048434033965654603223172380 5...

C(2n+1,n)*(n + 1)^2 (OEIS A002544)

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)^2 \binom{2n+1}{n}} = \frac{\pi^2}{9}$$

1.09662271123215095764827677776401679281263326747119895849 0... (OEIS A100044)

$$\sum_{n=0}^{\infty} \frac{(-1)^{n}}{(n+1)^2 \binom{2n+1}{n}} = 4\sinh^{-1}\left(\frac{1}{2}\right)^2$$

0.92625928230877756998771628492613104006562540019719548326 1...

C(2n,n)^2 (OEIS A002894)

$$\sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}^2} = {}_3F_2\left(1, 1, 1; \frac{1}{2}, \frac{1}{2}; \frac{1}{16}\right)$$

1.280498869108735691043469494574575659109327650536011481901...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\binom{2n}{n}^2} = {}_3F_2\left(1, 1, 1; \frac{1}{2}, \frac{1}{2}; -\frac{1}{16}\right)$$

0.775467204380406225054751283477724162453961837918964445723...

(2*n + 1)*C(2n,n)^2 (OEIS A000515)

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)\binom{2n}{n}^2} = {}_3F_2\left(1, 1, 1; \frac{1}{2}, \frac{3}{2}; \frac{1}{16}\right)$$

1.089270235168395517038258999660462839895436065220832214357...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\binom{2n}{n}^2} = {}_3F_2\left(1, 1, 1; \frac{1}{2}, \frac{3}{2}; -\frac{1}{16}\right)$$

0.921886408324695762275107416705968938745043502817821943330...

C(4n,n)/(3n + 1) (OEIS A002293)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{\binom{4n}{n}}{3n+1}} = {}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{27}{256}\right) + \frac{3}{4}\,{}_4F_3\left(\frac{4}{3}, \frac{5}{3}, 2, 2; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; \frac{27}{256}\right)$$

2.303791853815356015864186727403139946805009501831179591572...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{\binom{4n}{n}}{3n+1}} = {}_4F_3\left(\frac{1}{3}, \frac{2}{3}, 1, 1; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{27}{256}\right) - \frac{3}{4}\,{}_4F_3\left(\frac{4}{3}, \frac{5}{3}, 2, 2; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{27}{256}\right)$$

0.210781014679201975210523825404860121669311814511590683169...

C(5n,n)/(4n + 1) (OEIS A002294)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{\binom{5n}{n}}{4n+1}} = \frac{1}{5}\left(5\,{}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}; \frac{256}{3125}\right) + 4\,{}_5F_4\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 2; \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}; \frac{256}{3125}\right)\right)$$

2.232522348745708071115814253229776705280251664271500146744...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{\binom{5n}{n}}{4n+1}} = {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}; -\frac{256}{3125}\right) - \frac{4}{5}\,{}_5F_4\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 2; \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}; -\frac{256}{3125}\right)$$

0.174580268353169028788049411885297302867644130842643111192...

C(6n,n)/(5n + 1) (OEIS A002295)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{\binom{6n}{n}}{5n+1}} = {}_6F_5\left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1; \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}; \frac{3125}{46656}\right) + \frac{5}{6}\,{}_6F_5\left(\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2, 2; \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}; \frac{3125}{46656}\right)$$

2.188450610744303584862765744689120609633101993781600786622...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{\binom{6n}{n}}{5n+1}} = {}_6F_5\left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1; \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}; -\frac{3125}{46656}\right) - \frac{5}{6}\,{}_6F_5\left(\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2, 2; \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}; -\frac{3125}{46656}\right)$$

0.148867341339521072872366399710812337633369939750455820170...

C(7n,n)/(6n + 1) (OEIS A002296)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{\binom{7n}{n}}{6n+1}} = {}_7F_6\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, 1; \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}; \frac{46656}{823543}\right) + \frac{6}{7}\,{}_7F_6\left(\frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2, 2; \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \frac{12}{7}, \frac{13}{7}; \frac{46656}{823543}\right)$$

2.158466941948927152591414481806028692081524564658772159318...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{\binom{7n}{n}}{6n+1}} = {}_7F_6\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, 1; \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}; -\frac{46656}{823543}\right) - \frac{6}{7}\,{}_7F_6\left(\frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2, 2; \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \frac{12}{7}, \frac{13}{7}; -\frac{46656}{823543}\right)$$

0.129703522067233617745312452697533196838212761882723872639...

n*C(n+2,3) (OEIS A002417)

$$\sum_{n=1}^{\infty} \frac{1}{n\binom{n+2}{3}} = \frac{1}{4}\left(2\pi^2 - 15\right)$$

1.184802200544679309417245499938075567656849703620395313206...

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\binom{n+2}{3}} = \frac{1}{4}\left(27 + \pi^2 - 48\log(2)\right)$$

0.899634933552995941701837292470918966922423239487134607155...

4*C(2n+3,n)/(n + 4) (OEIS A002057)

$$\sum_{n=0}^{\infty} \frac{1}{\frac{4\binom{2n+3}{n}}{n+4}} = {}_3F_2\left(1, 1, 4; 2, \frac{5}{2}; \frac{1}{4}\right) - \frac{5}{12} + \frac{5\pi}{18\sqrt{3}}$$

1.350766631346345436144115458757897540682448124894041143087...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\frac{4\binom{2n+3}{n}}{n+4}} = \frac{3}{4}\,{}_3F_2\left(1, 1, 4; 2, \frac{5}{2}; -\frac{1}{4}\right) + \frac{7}{25} - \frac{12\sinh^{-1}\left(\frac{1}{2}\right)}{25\sqrt{5}}$$

0.805296723928254782335325632599215857053928698097460136410...

C(4n+1,2n) (OEIS A002458)

$$\sum_{n=0}^{\infty} \frac{1}{\binom{4n+1}{2n}} = {}_3F_2\left(1, 1, \frac{3}{2}; \frac{3}{4}, \frac{5}{4}; \frac{1}{16}\right)$$

1.108563435104316693465568461545165666232954105925613767584...

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\binom{4n+1}{2n}} = {}_3F_2\left(1, 1, \frac{3}{2}; \frac{3}{4}, \frac{5}{4}; -\frac{1}{16}\right)$$

0.907392237753950098823828925983851494486255990266306682106...

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